Modelling of Three–Dimensional Field Around Lightning Rods

By: Ruth Whitaker

November 2001

University of Tasmania

School of Engineering

Supervisor: Dr. D.J.H. Lewis

Submitted in partial fulfilment of the requirements for the degree of Bachelor of Engineering with Honours

ABSTRACT

This project aimed to model the three-dimensional field around a lightning rod prior to a lightning strike. The project is largely theoretical and discusses the various processes that have to be included to accurately model the voltage distribution and the charge density around the tip of the rod.

A program has been written in C++ to model the potential distribution around a lightning rod in three-dimensions. This program provides a framework for the solution of the three-dimensional problem.

ACKNOWLEDGEMENTS

I would like to thank Dr. Lewis for all his help and enthusiasm.

I would also like to thank Jonathan Burgess for his support and patience.

CONTENTS

1. INTRODUCTION

As our society becomes increasingly reliant on electronically controlled systems, protection of these systems from damage and failure also becomes more important. Lightning can cause injury to people and animals, as well as damage to buildings and equipment. Lightning protection for individual pieces of sensitive equipment and mains power can be achieved by making use of lightning arresters and surge protection equipment. Lightning protection for buildings is most commonly achieved by using some form of lightning rod.

Benjamin Franklin was the first to propose experiments to prove that lightning was a form of electricity. Once the link between lightning and electricity had been established methods of lightning protection began to be investigated with a real chance of success. In 1753 Benjamin Franklin announced his lightning rod to the public as a means of lightning protection for all structures.

Lightning rods are still the predominant form of lightning protection for a wide range of structures and vessels. Lightning rods can often be seen on top of tall buildings in cities around the world.

Since Benjamin Franklin's time many improvements have been made to the lightning rod. However, lightning rods are still not perfect and the more we know about their operation the better our lightning protection systems will become.

In 2000 Benita Cridland and Kristian Batge undertook an honours project on the subject of "Modelling of Corona Effects Above Lightning Rods". This project aimed to model the electric field present in the pre-strike phase of a thunderstorm. My project is a continuation of their work. B. Cridland's thesis used the C++ programming language to write a program to model the electric field around the tip of the rod in two dimensions by assuming axial symmetry. B. Cridland's model took into account the generation, recombination and travel of charge.

The project also investigated the effects on the electric field when rods of different geometries were used. This included varying the tip height of the rod, and the radius of curvature i.e. the sharpness, of the point.

The effect of various geometries on the formation of the space charge layer and the net intensification factor were also investigated.

It was found that although a sharper tip increased the initial electric field, once corona effects were accounted for the peak electric field was of a similar strength for the various geometries tested (3.17 – 3.22 MVm^{-1}).

If the radius of curvature of the rod was greater, i.e. the rod was blunter, the electric field intensification effects were maintained at greater distances from the rod. This suggests that a blunter rod would be better at maintaining an upward streamer.

It was concluded that the most effective lightning rod is one which is blunter, whilst still being sharp enough to ensure that the ambient electric field is intensified to a value at which ionisation may occur.

However, B. Cridland did not account for any environmental conditions such as wind, which will almost certainly affect the processes involved. So that the effect of wind can be modelled the system first needs to be modelled in three dimensions. This thesis aims to model the field around the rod in three dimensions so that the effect of wind can be investigated.

This project was largely theoretical and to give a background there is a general introduction to thunderstorms and lightning in Section 2. The life cycle of the thundercloud is described and theories about how the cloud becomes charged are discussed in Sections 2.1 and 2.2. There is an outline of the processes that occurs during a lightning strike in Section 2.3.

To outline some of the important principles of a lightning rod a brief overview of the essential elements of a lightning rod has been included in Section 3. There is also a description of the process by which a lightning rod attracts lightning to divert it from the building it is protecting.

Corona and space charge are crucial processes when studying the effectiveness of a lightning rod. In order to understand the launching and sustaining of upwards leaders the processes of corona and space charge must first be fully understood. In Section 4.1 there is an outline of the process of corona emission from the tip of a lightning rod. There is a brief discussion on the effects of space charge on the electric field surrounding a lightning rod in Section 4.2.

Another important consideration when determining the charge density around a lightning rod is the movement of the charge species. There are two main processes that cause the travel of charge species in an electric field. They are mobility and diffusion and they are discussed in Section 5.

When determining the behaviour of charged species in the region around a lightning rod we must understand the processes by which these species are generated and the processes that cause their decay. A general description of the various processes by which ions are generated in the electric field around the rod is provided in Section 6.1. It includes ionisation by collision and secondary effects. In Section 6.2 there is a discussion of the various mechanisms by which ions decay.

Although there are a number of processes outlined there are really only a few of these processes that have significant effects on the charge density. Last years model only included ionisation by collision and recombination. It is proposed that deionisation by attachment is also a process which should be included in the model.

In order to model the electric field around the rod Laplace's Equation was applied iteratively to calculate the final voltage distribution. The mathematics that was used to model the voltage distribution in the field around the rod is given in Section 7.

It is important to understand the operation of the program and its limitations if further work is to be done on this project. An outline of the program has been included in

Section 8 so that the reader can gain an understanding of its operation without looking at the code.

This project can obviously be further developed now that the basis for the three dimensional problem is in place. Possible avenues for further work on this project are discussed in Section 9. The charge density can be included and the generation, decay and movement of the charge species can be modelled. There are also some different approaches that may be used to solve this problem.

The model was not entirely accurate; a discussion of the results is given in Section 10.

The framework for the three dimensional problem is in place. Although the program does not correctly model the electric field around the rod it is very close. An error in the boundary conditions has lead to the electric field being distorted around the tip.

2. THE THUNDERCLOUD AND THE LIGHTNING STRIKE

2.1. The Thundercloud

In order for a thundercloud to form the air must be moist and unstable, and there must be sun heating the earth and the air close to the ground for at least part of the day.[1]

The heat from the sun warms the earth, which in turn heats the air near the ground. This warm air near the surface of the earth has a tendency to rise and cooler air flows in under it. The warm air rises quickly and expands when it reaches higher altitudes. As the air rises into the colder upper atmosphere it starts to cool down. As the air cools the moisture which it has carried from the ground begins to condense, forming a cloud. Under normal circumstances the rate of formation of a cloud is matched by the rate of dissipation as the surrounding cool dry air absorbs the condensation as quickly as it forms.

However in a thundercloud the condensation forms more quickly than the surrounding air can absorb the condensation. For a thundercloud to form it is also usual for the surrounding air to be moist and therefore less able to absorb the condensation when it forms. Because the surrounding air cannot absorb the condensation as quickly as the thundercloud produces it, the cloud continues to grow with the warm updraft of air from the ground and becomes a thundercloud. The changing characteristics of a thunderstorm can be divided into three separate stages.[2]

2.1.1. The Cumulus stage

During the cumulus stage there are usually strong updrafts throughout the forming thundercloud. In the early stages a strong updraft rises into the air forming a giant column of cloud as discussed above. The warmer the air near the surface of the ground the faster the updraft is and this means that more moisture is carried upwards and it is carried higher.

As the warm air rises into the colder upper atmosphere it starts to cool down and the moisture begins to condense. This results in large numbers of very small water droplets being formed, which is what forms the cloud. The relative humidity of the surrounding air must be high, close to 100%, so the evaporation of the water droplets in the cloud is small. The high relative humidity of the surrounding air means that the water vapour condenses more quickly than the surrounding air can absorb it. Also as the vapour condenses heat is released, this warms the air slightly and adds to the updraft which continues to feed the formation of the cloud.

As the cloud continues to grow higher the water vapour in the highest regions becomes very cold and it begins to freeze, forming small ice particles. Condensation from the lower levels continues to be carried upwards into these colder regions. As the condensation hits the newly formed ice crystals it freezes, which results in slightly larger ice particles, which eventually become hailstones. Meanwhile the water droplets in the lower regions of the cloud begin to join together and form raindrops.

The raindrops and the hailstones continue to grow until they become too large to be supported by the updraft, when this stage is reached the rain and the hail starts to fall. Usually the hailstones melt as they fall through the lower warmer regions of the atmosphere and become rain. However if the lower regions are sufficiently cooled the ice particles do not melt and they fall as hail. Once the rain begins to fall the thunderstorm enters its mature stage.

2.1.2. The Mature stage

During the mature stage of a thunderstorm there are updrafts and downdrafts in the cloud. The rain has started falling and it is usually accompanied by lightning. The weight of the falling rain exerts a downward force on the updraft of warm air that is still rising from the ground. This downward force is usually only enough to create a downdraft in the lower part of the cloud, but the updraft in the upper part of the cloud is only slowed. Therefore the updraft is not completely halted by the downdraft cause by the rain.

The downdraft creates strong gusty winds, which hit the ground and travel outwards from the storm in all directions. These winds are often very strong and can cause considerable damage to houses and bushland. The temperature near the surface preceding a thunderstorm is usually quite high. However the downdraft brings cooler air and rain, and this results in a sudden drop in temperature at ground level.

The rain usually only lasts for about 15 to 30 minutes and during this time the downdraft spreads throughout the cloud. As the downdraft continues to spread it gathers enough force to lessen the updraft considerably. This means that there is no longer a large amount of warm air being carried upwards and the cloud's growth starts to slow down. Eventually the downdraft spreads throughout the entire cell and the cloud begins to enter its final phase, the dissipating stage.

2.1.3. The Dissipating stage

A thundercloud is close to dissipating when over half of the cell is made up of downdrafts.[3] The downdraft brings colder air from the upper atmosphere to ground level which means that the air near the ground becomes cooler and therefore the updraft, which was created by the warm air rising, weakens and less moisture is carried upwards. This results in less condensation, and less rain and hail which weakens the downdraft. As the downdraft lessens the wind speeds decrease and the intensity of the rain decreases. There are no longer any large raindrops in the cloud, but a light drizzle may continue to fall until the cloud breaks up.

2.2. Charging a Thundercloud

There are several theories about how the thundercloud becomes charged as has been observed. Most theories agree that the strong updraft present in a thundercloud is the most likely cause of charge separation. How the particles become charged is a source of contention.

The theory of "ionic charging" was first proposed by C. T. R. Wilson.[4] Wilson proposed that larger droplets of water could capture negative ions which remained in the lower regions of the cloud, while the positive ions were carried to the upper regions of the cloud by the updraft.

Experiments performed at the New Mexico Institute of Mining and Technology in the fifties, by E. J. Workman and his colleagues Stephen E. Reynolds and Marx Brook, found that when two pieces of ice of different temperatures collide, charge is separated. They discovered that the warmer particle takes on a negative charge, and the colder particle takes on a positive charge[5]. In a thunderstorm the small particles of hail present would be warmer than the surrounding ice crystals. Therefore when they collided the hail would become negatively charged and the ice crystal would become positively charged. The larger heavier hailstones would fall to the lower regions of the cloud, and the small ice crystals would be carried to the higher regions of the cloud by the updraft.

Although there is no one theory that adequately describes the charging process of a thundercloud, experiments and observation "support the idea that ice particles play an important role in the formation of thundercloud electrification"[6]

However there are cases of thunderstorms and lightning where it is very unlikely that ice particles would be present. These cases can be found in tropical regions where thunderstorms are frequent but the atmosphere is warm and moist, therefore the presence of ice crystals is improbable.

It has also been proposed that the turbulent air in the thundercloud leads to friction between particles, which produces charged particles. Larger particles are most likely to become negatively charged and smaller particles positively charged. There have been cases of lightning occurring in the ash cloud of a volcano. These cases illustrate that lightning can occur as a result of particles being charged by friction.

With a number of possible theories, we are lead to suppose that thunderclouds can be charged by a number of different methods.

Although there is no theory that fully explains how a thundercloud becomes charged we do know what the resulting charge on the cloud is. Thunderclouds have two major regions of charge. The lower part of the cloud has a predominantly negative charge and the upper portion of the cloud has a predominantly positive charge. When the rain begins to fall a small region of positive charge often forms near the base of the cloud.

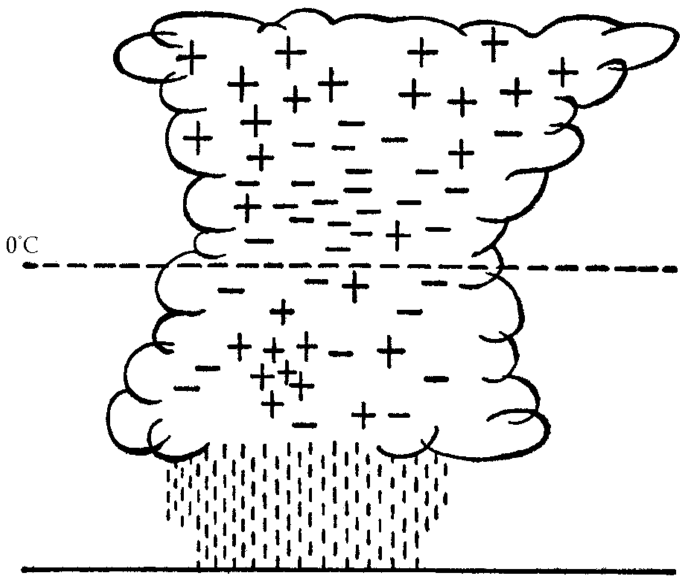


Figure 2.2.1: Charged regions of a thundercloud.[7]

2.3. The Lightning Strike

Lightning occurs when a region in the atmosphere attains a charge large enough to result in the electrical breakdown of air. The most common producer of lightning is the thundercloud.

There are two main types of lightning strokes, cloud–to-cloud, and cloud-to-ground. Obviously we are mainly interested in cloud–to-ground strokes because they can cause destruction and sometimes death. However cloud–to-cloud strokes are also important because of the potential for them to strike aeroplanes and other airborne devices e.g. weather balloons, surveillance craft etc.

In most cases lightning strikes bring negative charge to earth although there are rare cases of positive discharges.

To understand the lightning strike we must first understand the conditions that exist prior to the strike.

Under normal circumstances the earth has a negative charge relative to the positively charged ionosphere. However in the presence of a thundercloud this field is reversed. The base of a thundercloud has a large negative charge that repels electrons on the earth's surface resulting in the earth being positively charged with respect to the cloud.

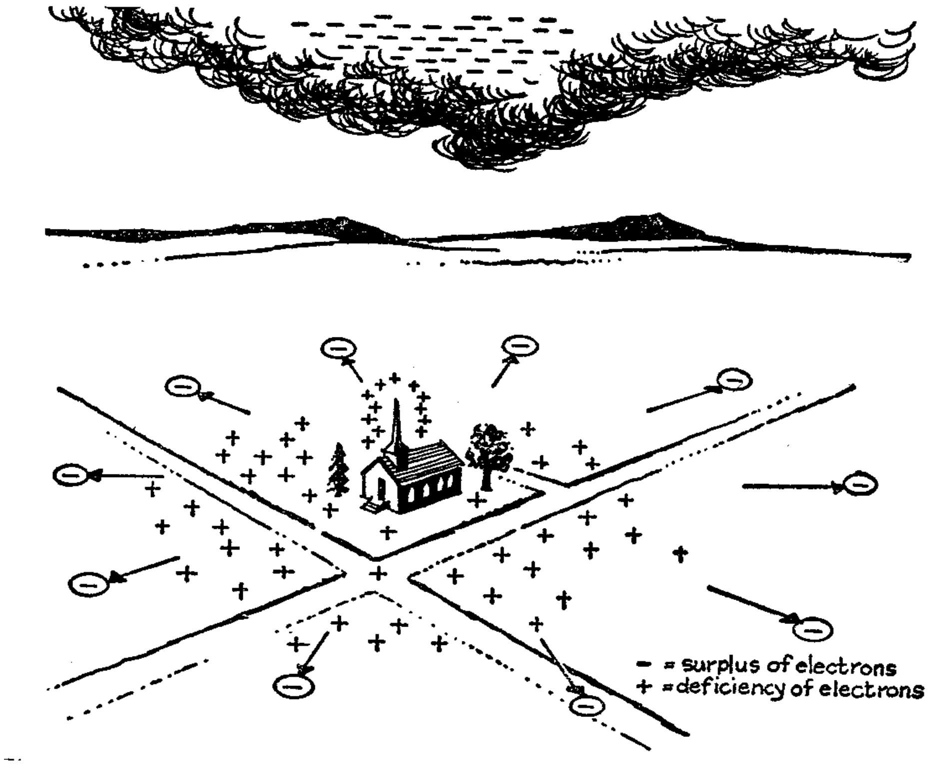


Figure 2.3.1: Prior to a lightning strike the base of the cloud has a large negative charge, and there is a positive charge induced on the ground below.[8]

This means that electrons flow into points on the earth's surface in an attempt to equalise the potential difference. Points become more positively charged than the rest of the surface because of their geometry and if the potential difference is great enough a bluish corona glow may be visible. This corona glow is due to the ionisation of the air around the point, also known as point discharge. Although the process is called point discharge, electrons are actually flowing into the point.

Initially the air acts as an insulator between the thundercloud and the ground. As the storm grows, so does the potential difference between the cloud and the ground. The negatively charged base of the cloud exerts a strong downwards force on the free electrons in the air. This force accelerates the electrons, and if they reach sufficient speeds they can cause ionisation upon collision with other particles in the air. When particles are ionised by collision with an electron a positive ion and an electron are created.

Ionised air is a good conductor of electricity and electrons in the thundercloud move towards this region aiming to find a quicker path to ground. This initiates a downward leader, known as a stepped leader.

The field in front of the tip of the leader is very intense and this causes electrons the rapidly multiply by ionisation by collision.[9] This strong field causes successive electron avalanches, which cause the stepped leader to move downwards in steps.

As the stepped leader approaches the ground the air near the ground also becomes ionised and streamers rise from the earth to meet the leader. In the case of tall trees or buildings the downward leader has less distance to travel before it joins with the upward streamer. That is why tall objects are more likely to be struck by lightning.

This is the basis of a lightning rod. The streamer from the rod is higher than any streamers that may be rising from the building, which it is protecting. Therefore the lightning is attracted to the rod, and not the building beneath it. The ability of a lightning rod to create and sustain an upwards streamer is crucial to its being an effective form of lightning protection.

When the downward leader and the upward streamer meet there exists an ionised path from cloud to ground. The electrons from the downward leader rush down the last Section of the path, the upwards streamer, to the ground. This first rush of electrons is known as the “lightning prestrike” and it causes very high currents of a brief duration.[10]

The electrons in the ionised path flow to the ground starting from the bottom and moving upwards. This movement of electrons brightly illuminates the channel from the earth to the cloud and is known as the return stroke. The lightning strike we see is this flow of electrons from the ionised path to ground. When lightning strikes the current builds rapidly to a peak, then drops of slowly. It is this initial rapid rise in current that causes the most damage during a lightning strike.

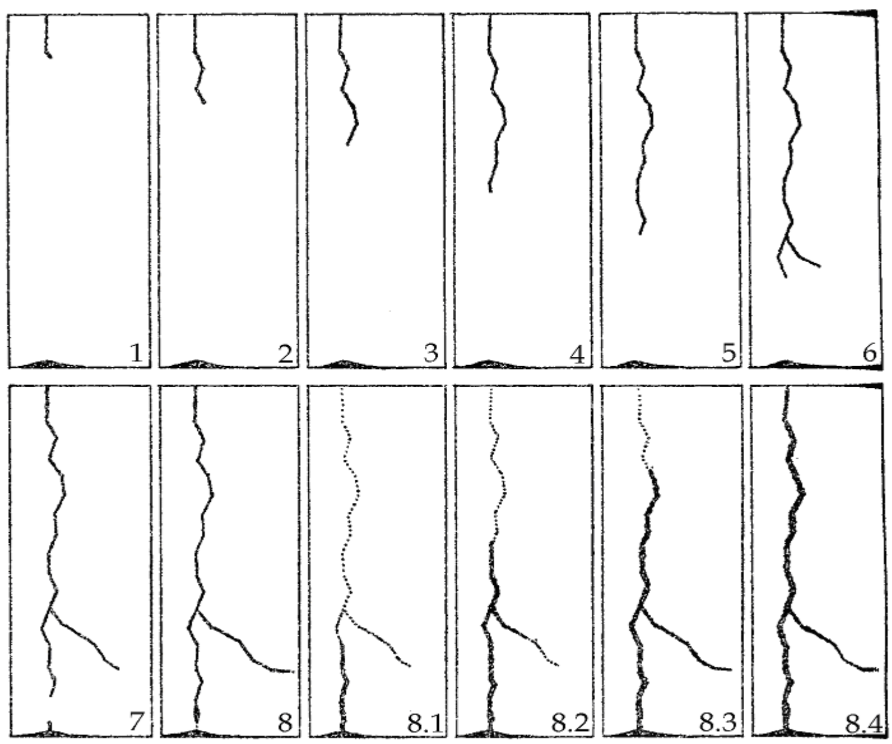


Figure 2.3.2: Frames1 to 8 show advance of stepped leader, frames 8.1 to 8.4 show fast return stroke.[11]

It was shown that in 80% of lightning strikes to the Empire State Building the stepped leader was initiated from the building, not the cloud.[12] The stepped leader is only initiated from the ground in cases of unusually tall buildings, mountains, or other isolated objects. However there is still a final leader that travels from the cloud to the ground and the return stroke moves up the channel in the same way as when the leader is initiated from the cloud.

After the first return stroke although most of the charge has been drained from the channel some still remains. This means that the path is still a good conductor for about 0.1 seconds after the return stroke. This is because any charge remaining in the channel starts to recombine as soon as the current becomes small, and after 0.1 seconds the ions have recombined to such as extent that an ionised path no longer exists.[13] Before the charge has had time to sufficiently recombine there is often a second stroke using the same path. In most cases a lightning strike is made up of multiple return strokes. In this case there does not have to be a stepped leader because the path to ground is already ionised, instead there is a dart leader. The dart leader travels down the path quickly and another return stroke follows which discharges more negative charge from the cloud.

There are usually multiple strokes because each stroke drains current from a different region of the cloud. It is now possible for another lightning stroke to occur between a region in the cloud that is still charged and the now relatively uncharged region where the first stroke originated. The electrons from this stroke find the previously ionised path and a dart leader heads to the ground, and there is another return stroke. There may be typically three of four return strokes as different regions of the cloud discharge. Although there may be a number of return strokes it appears to the human eye as a single flash because the entire process occurs so quickly.

The stepping of the downward leader is due to a series of corona bursts. At first free electrons in the air create positive ions and electrons by ionisation by collision. These electrons lead to further ionisation and a small streamer develops. The electrons from the cloud surge down the short, ionised channel. When they reach the end of the channel they flare into a pestle shaped burst.[14] By a random process some electrons create an ionised path more quickly than the others, and the other electrons join this new path. When all the electrons have joined the newly ionised path the pestle shaped corona burst collapses and the electrons surge down the ionised path until they reach the end and form a new corona burst. This process is repeated until there exists an ionised path to ground, and it accounts for the stepped nature of the leader.

2.3.1. Different Types of Lightning

Some lightning strikes look quite different from others even though electrically they are the all caused by the same process. There are a number of types of lightning.
Streak lightning is the most common form of lightning and it appears as a continuous streak of light from the cloud to the ground. Sometimes the lightning branches and produces multiple paths to the ground. This is known as forked lightning.[15]

Heat lightning is lightning which results in the illumination of the cloud, but is not accompanied by thunder. Heat lightning occurs when the lightning is so far away that the thunder cannot be heard, but the light can be seen. The light from the flash can be scattered or reflected by the clouds, and can result in large areas of cloud being illuminated.[16]

Sheet lightning is caused by intracloud lightning that cannot be seen because of the clouds. The cloud droplets and ice crystals and cloud droplets also diffuse the light so that lightning illuminates a large area of the thunderclouds. Sheet lightning is usually accompanied by thunder, however sometimes it is silent. Although sheet lightning does not look the same as a lightning flash between to separate clouds with clear sky between them, the electrical process is the same.[17]

Ribbon lightning occurs when there is a multiple flash stroke in which the individual flashes appear to be shifted sideways. This displacement is due to strong winds. In a strong wind the ionised path of the lightning strike can be blown sideways between return strokes, resulting in a number of luminous paths.[18]

A 50 km/h wind would move the air horizontally at 13.89 m/s. In a multiple flash stroke the second flash may occur about 0.07 seconds after the first stroke. Therefore the second stroke could be shifted about 0.97 m from the first stroke.

This phenomenon supports the further investigation of wind effects on the effectiveness of lightning rods.

Bead lightning is more unusual and there are a number of theories as to its exact cause. Initially it appears as a luminous path from cloud to ground, however instead of the entire path disappearing at the same time the luminous path dissolves into bright portions which give the appearance of a string of beads. Bead lightning occurs relatively infrequently.[19]

The most rare and unusual type of lightning is ball lightning. Ball lightning is a luminous ball that “floats” through the air normally quite close to the ground. The typical size of ball lightning is about 20 cm diameter. Ball lightning occurs in the presence of thunderstorms and so we must look to the electric charge associated with a thunderstorm to explain the phenomenon. There are a number of theories, but there is no comprehensive explanation for the phenomenon of ball lightning.[20],[21]

3. LIGHTNING RODS

Early man regarded lightning as a power of the gods. It was not until 1752 that this myth was dispelled for good when Benjamin Franklin in the USA and T.F. D'Alibard in Europe proved that lightning was electricity being discharged from clouds.[22]

The protection of structures from lightning using the lightning rod had been suggested before this but it was not until 1753 that Franklin outlined the basic features of a lightning rod to the public in a paragraph entitled, "How to Secure Houses, &c. from Lightning". Franklin wrote,

> "...provide a small iron rod ... of such length, that one end being three or four feet in the moist ground, the other may be six or eight feet above the highest part of the building. To the upper end of the rod fasten about a foot of brass wire ... sharpened to a fine point ... A house thus furnished will not be damaged by lightning, it being attracted by the points, and passing through the metal into the ground without hurting any thing."[23]

Franklin noticed that objects such as trees and church steeples were favourite targets for lightning and he hypothesised that this was because of their height. Therefore he proposed that a tall metal rod erected on top of a building would also attract lightning.

The rod attracts lightning because it has the same potential as the ground, however the potential of the surrounding air increases with height. Therefore the higher the rod is the greater the potential difference between the tip of the rod and the surrounding air. The potential difference required for ionisation to occur is that for the breakdown of air, which is approximately $3x10^6$ volts/m.

Once the lightning had been attracted to the lightning rod it could then be diverted safely to ground without damaging the structure. This was achieved by attaching conductors to the lightning rod, and connecting the other end of the conductors to rods that were driven into the ground. This provided a path for the current to flow through the lightning rod and the conductors to the earth.

Lightning rods are widely used to protect buildings and the principle is the same as the rods that Franklin first proposed. However we know have a far better understanding of lightning strikes. Strikes to ground and aircraft have been measured and described in detail and lightning is well understood as an electrical phenomenon.

The parameters that are important in the design of lightning protection systems have also been accurately measured. For example: the peak current (I), the peak rate of rise of current (di/dt), the current transferred in a strike ($\int idt$), the action integral ($\int i^2 dt$), the duration of pulses, the number of pulses etc.[24]

The action integral is important when determining the amount of energy that can be discharged into a certain load.

$Energy = R \times \int i^2 dt$ Joules, where R is a fixed resistor, in the case of lightning it is a constant. Lightning is a constant current source. Therefore the load it flows through e.g. building, tree etc., has little effect on the current waveform.

The electric fields that precede lightning strikes have also been measured and recorded

Some important parameters of lightning measured at the ground various strokes are:[25]

	Severe	Average
I	200 kA	20 kA
di/dt	150 kA/μs	30 kA/μs
$\int idt$	200 coulombs	15 coulombs
$\int i^2 dt$	2×10^6 A^{2}s	0.07×10^6 A^{2}s

Lightning rods divert lightning from a building by providing an easier path to ground than a path through the building or surrounding air. Lightning rods do not provide complete protection and lightning can strike the side of a tall building even if it protected by lightning rods. However if the rod is properly installed it reduces the risk of being hit by a direct strike to nearly zero.

There are some key elements to consider when using lightning rods to protect a building.

- The lightning rod must be made from a good conducting material and must be positioned to adequately protect the building, more than one rod can be used.
- The conductors connected from ground to the rod must be of sufficient size to withstand the currents resulting from a lightning strike.
- The rod must be properly grounded otherwise it may cause more harm than if it wasn't there at all.

If a lightning rod is in place, but not properly grounded it can cause considerable damage. The lightning rod will attract lightning that may not have otherwise struck the building, and the down leads will conduct it to ground. If the earth is not low resistance the electricity will find a lower resistance path to ground, usually through the building we are trying to protect. The high currents can jump from the down leads to wiring in the building and cause extensive damage.

There are three essential elements in a lightning protection system:[26]

1. The air terminal, the part that actually gets struck,
2. The down lead, the conductors that carry the current to earth,
3. The earth termination.

There are a number of different air terminations that can be used, however this chapter will concentrate on the Franklin lightning rod.

A lightning rod should be positioned such that it is in the region where the highest electric field will develop. It must provide a low resistance path to ground for the lightning. The most important function of a lightning rod is that when the leader from the cloud approaches it can initiate and sustain an upwards streamer to meet the leader from the cloud. The current can then flow through the conductors to ground.

The ionised region around the tip of a rod is usually quite small. However when there is a lightning stroke approaching this the rate of ionisation around the tip increases, and an upwards streamer forms to reach towards the downwards leader. This is because the stepped leader has a strong electric field around it that increases the electric field in the air, which increases the rate of ionisation. This upward streamer can effectively double the height of the rod.

The probability of a building being hit by lightning increases with height. Therefore most tall buildings have some form of lightning protection. The size of the building will also impact on the likelihood of being struck, but this becomes less significant as the height increases.

Tall buildings or trees will protect shorter objects in their immediate vicinity from being struck by lightning. As a rule of thumb a lightning rod will provide a "cone of protection", defined by the following diagram:

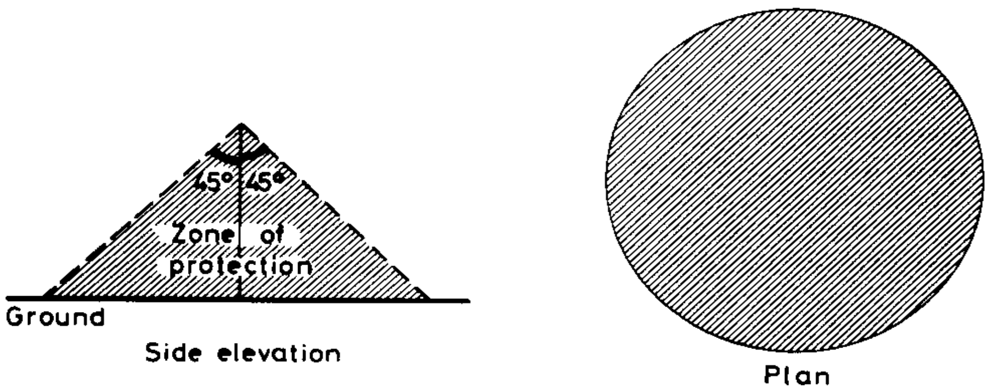


Figure 3.1: The circular right cone of protection afforded by a mast. Objects within the cone are said to be protected.[27]

However this does not hold for buildings which are taller than, say 30m, and there are a large number of documented cases of tall buildings being struck on their sides.[28] This is known as side flash and is responsible for most of the cases in which a protected building has been damaged by lightning.[29]

A better definition for the protection provided by a lightning rod can be derived using the rolling sphere method. However to use this method we must first define the "striking distance". Consider the stepped leader that travels downwards in a series of random steps and branches. When it approaches the ground to within 20 – 200 m the building launches a streamer to meet the stepped leader. The distance of the downward leader from the building when the streamer is launched is called the striking distance.

Using the rolling sphere method the cone of protection is defined as "the volume untouched by a sphere of radius equal to the striking distance, rolling around the tower or building". This is represented diagrammatically below:

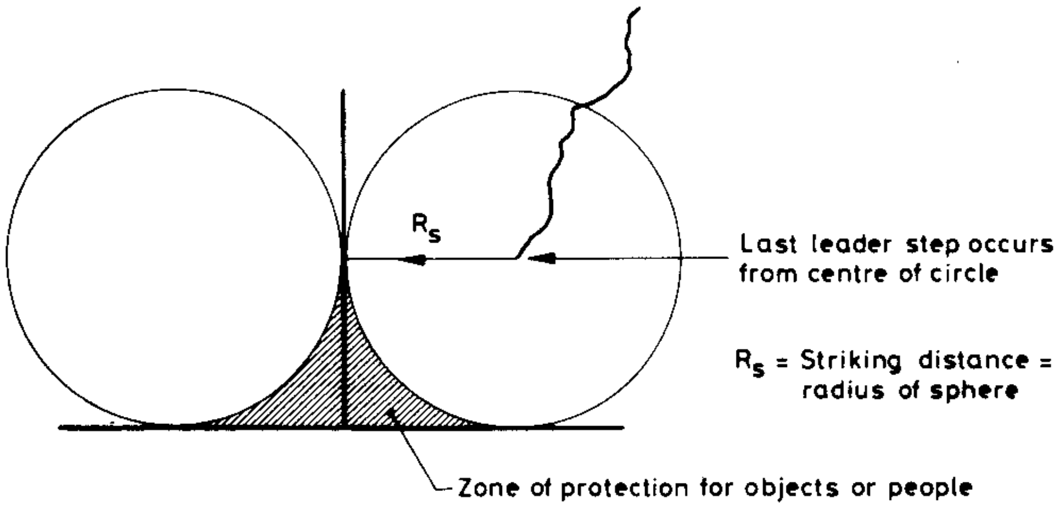


Figure 3.2: Rolling sphere method of describing the zone of protection afforded by a lightning rod. Successfully predicts that tall buildings can be struck well down from the top.[30]

The ability of a lightning rod to initiate and sustain an upwards leader depends on a number of factors.

A properly installed lightning rod can provide very good lightning protection for all kinds of structures. However, lightning can still cause damage inside a building by travelling through the mains power supply.

Transmission lines are protected from direct lightning strikes by using an overhead ground wire, which is earthed at appropriate intervals and lightning arresters. They are also protected from induced currents from nearby lightning strikes by the use of lightning arresters.

Lightning arresters consist of an air filled gap that provides a high resistance to normal currents, but when a large current induced current flows the air in the gap is ionised and the current has a low resistance path to ground.

The down leads and ground termination must also be properly installed for the lightning protection system to provide good protection.

4. CORONA AND SPACE CHARGE

4.1. Corona

Corona is "the rarefied gaseous envelope of the sun and other stars. The sun's corona is normally visible only during a total solar eclipse, when it is seen as an irregularly shaped pearly glow surrounding the darkened disc of the moon."[31] The term corona is also used to describe "the glow around a conductor at high potential"[32]. It is the second definition of corona that is relevant to this thesis.

Consider a thunderstorm above the earth. The base of the thundercloud has a large negative charge, which induces a positive charge at the ground, especially on tall objects. An electric field exists between the cloud and the ground.

When a lightning rod is placed in this electric field, the electric field strength near the tip is much larger than elsewhere. This is because the electrostatic field lines must enter a conductive material perpendicular to its surface, and thus the field lines are compressed at a point.

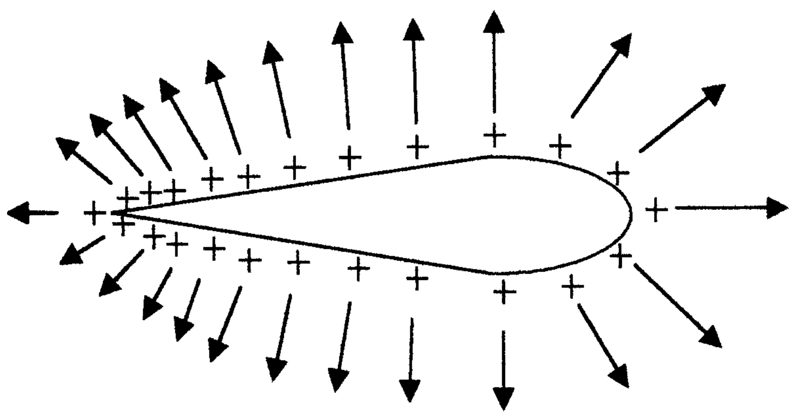


Figure 4.1.1: Positively charged pointed conductor showing compression of field lines around point.

In the strong electric field near the point a force $F = eE$ acts on the free electrons. Because the tip of the rod has a positive potential, free electrons are accelerated towards the point and cause ionisation by collision to occur. The electrons freed by the ionisation also rush towards the tip, and ionise more molecules. This process is known as an electron avalanche. This creates an ionised region which is a good conductor of electricity, therefore effectively increasing the height of the rod.

These electron avalanches continue until the quantity of space charge present has reduced the electric field at the tip to a point where ionisation by collision no longer occurs. Since electrons have a higher mobility than the positive ions the electrons move towards the tip quite quickly, but the positive ions drift away at a slower rate. The space charge causes a reduction of the electric field near the tip of the rod. Then as the positive ions starts to drift away the potential at the point begins to increase again. The potential continues to increase until ionisation by collision can start once more, and the whole process repeats itself. This is known as burst mode corona.

The positive ions produced form a space charge above the tip of the rod that decreases the electric field in the region, see Figure 4.1.2. Figure 4.1.2 only shows one component of the electric field, but it gives an indication of the effects due to space charge. The dotted line represents the original electric field and the solid line represents the distorted electric field due to the space charge. As can be seen in both cases the electric field increases closer to the point, however there is a decrease close to the tip of the rod when the effects of space charge are included.

The region of high electric field moves further away from the point over time, extending the region for ionisation. The field strength at the tip of the space charge may result in the initiation of a streamer.

The region of positive ions creates a new positive anode above the tip of the rod and the free electrons are now accelerated towards this new anode. This region of positive space charge creates an extension of the tip of the rod. This extends the region of ionisation towards the cloud above.

Apart from ionisation of surrounding molecules, oxygen and nitrogen molecules will also be raised to higher energy states through excitation by collisions with free electrons. When these molecules return to lower energy levels light is emitted, and depending on the energy levels the light may be visible to the human eye. This emitted light produces the coronal glow.

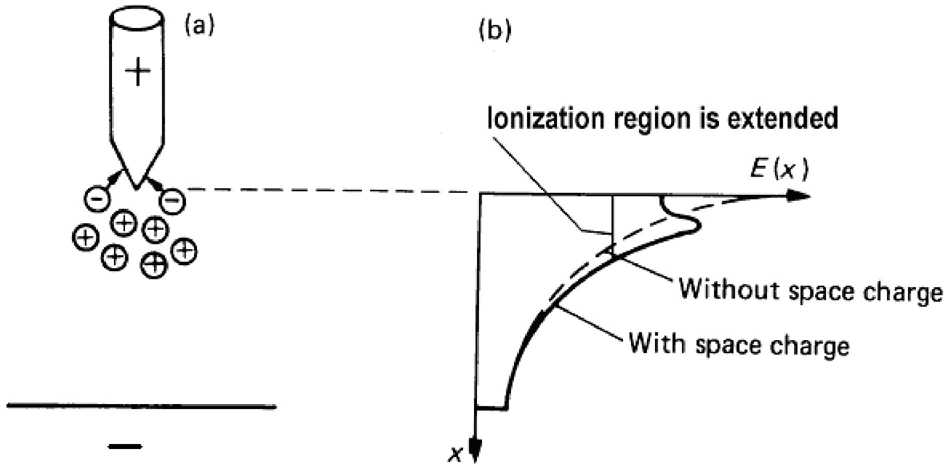


Figure 4.1.2: (a) Space charge build-up in positive point-plane gap. (b) Field distortion by space charge[33]

The value at which ionisation by collision begins is typically 500 kV/m for air (this is often referred to as the onset value). Corona may appear if field is close to this value.

If the electrical field is increased beyond the onset field, a streamer will form from the tip and, with each avalanche, grow in length. The streamer consists of ions through which electrons move towards the tip. The tip of the streamer acts as the new tip of the object, and avalanches now occur to this tip.

The streamer grows in length until the ambient electrical field is screened altogether by the leftover positive ions produced by the avalanches. The streamer will then be in a steady-state equilibrium.

4.2. Space charge

The field that is produced by the presence of considerable space charge is important in corona processes.

When considering a uniform electric field with strength $E = \frac{V}{d}$ the increase in the number of ions due to ionisation by collision is represented by $e^{\alpha d}$ (see Equation(6.1.1.2)). However this is only the case when the field due to the space charge present is negligible when compared with the electric field present.

While studying the effects of space charge Raether found that when the concentration of charge at the head of the avalanche was greater than 10^6 cm^{-3}, but less than 10^8cm^{-3} there was a decrease in avalanche growth.

$$\frac{dn}{dx} < e^{\alpha d} \quad [34] \qquad \text{Equation (4.2.1)}$$

However as the charge concentration increased beyond 10^8 cm^{-3} there was a steep rise in the measured current and eventually breakdown occurred. Raether attributed this steep rise in current and the weakening of the growth of the avalanche to the presence of space charge.

When the concentration of charge at the head of the avalanche exceeds 10^8 cm^{-3} this can lead to the initiation of a streamer.

The electric field created by a thunderstorm takes considerable time to build up. Over this period of time the products of ionisation have time to drift leading to the accumulation of space charge in the region above the rod and distortion of the original electric field.

As the electric field continues to increase the ionisation process undergoes a number of steps. At first the discharge appears as a small streamer with only a few branches. This streamer continues to grow in length and the number of branches also grows.

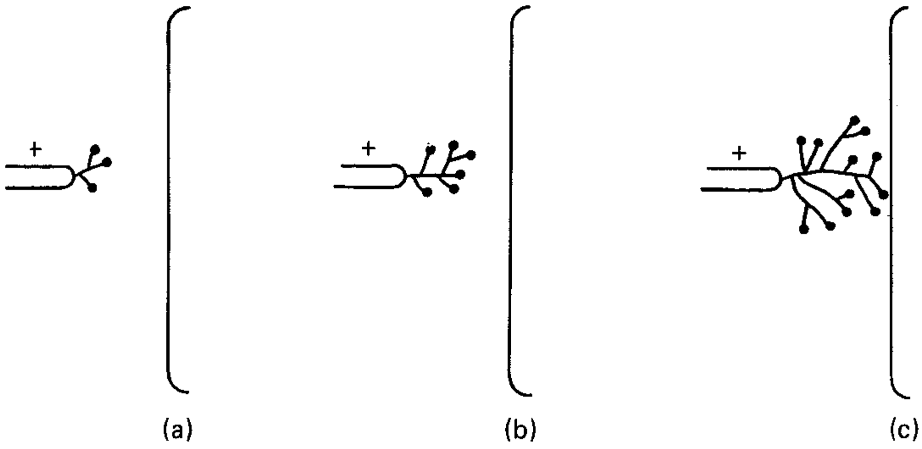


Figure 4.2.1: Diagram showing the formation of streamers between positive point and negative plane.[35]

The streamers grow in a series of bursts and this stage is called the onset stage. As the electric field continues to increase the streamers become more frequent and the discharge becomes self – sustaining. At this stage there is often a visible glow around the tip of the rod. The glow is usually a bluish colour and as the electric field is increased the intensity of the glow also increases.

5. MOVEMENT OF CHARGE

5.1. Mobility

In the presence of an electric field charged particles experience a force in the direction of the field given by:

$$F = eE \qquad \text{Equation (5.1.1)}$$

where e is the charge of the particle.

This causes charged particles to accelerate in an electric field. During this movement a charged particle collides with other particles and its velocity is kept to a constant value known as the drift velocity. The drift velocity in the direction of the field is defined as the mobility, K:

$$K = \frac{\mu}{E} (m^2 / V \sec) \qquad \text{Equation (5.1.2)}$$

where μ is the average drift velocity in the direction of the field and E is the electric field strength.

From this we can determine the current density, j. When there is a considerable amount of space charge the concentrations of the electrons and positive ions is not equal. Therefore we define the concentration of electrons and positive ions as n_i and n_e respectively. The corresponding current densities can be written as:

$$\begin{aligned} j_i &= n_i e K_i E \quad [36] \\ j_e &= n_e e K_e E \end{aligned} \qquad \text{Equation (5.1.3)}$$

5.2. Diffusion

In cases where the concentration of ions is non – uniform there is a movement of ions from regions of higher concentration to regions of lower concentration. This process is called diffusion. Together diffusion and mobility are largely responsible for the mass movement of charge in an electric field.

The movement of ions due to diffusion can be described in the following manner. Say that the concentration varies in the $x-$direction, then take a region perpendicular to this direction of unit area and thickness of dx. The number of particles crossing this area is proportional to the ion concentration gradient dn/dx. Therefore the flow of particles (flux) in the $x-$direction is given by:

$$\Gamma = -D\frac{dn}{dx}$$ Equation (5.2.1)

where D is a constant known as the diffusion coefficient.

The rate of change of concentration in the region defined is:

$$\frac{d}{dt}(ndx) = \Gamma - \left(\Gamma + \frac{d\Gamma}{dx}dx\right)$$
$$\frac{dn}{dt} = D\frac{d^2n}{dx^2}$$ Equation (5.2.2)

In three dimensions the above Equation becomes:

$$\frac{\partial n}{\partial t} = D\nabla^2 n$$ [37] Equation (5.2.3)

6. IONISATION AND DECAY PROCESSES

6.1. Ionisation Processes

Under normal conditions air is a good insulator, however if a large electric field is applied this is not necessarily the case. If the electric field is large enough charged particles in the air can gain sufficient energy so that when they collide with another particle they cause ionisation. For large electric fields ionisation by collision is the main process leading to the breakdown of air.

6.1.1. Primary Ionisation Processes - Townsend's first ionisation coefficient

Townsend used two parallel plates to investigate the effects of electric fields on gases. He found that at first the current increased proportionally to the applied electric field, until it reached a value where it remained almost constant. When the applied field was increased past this point the current began to increase again, this time the increase was exponential.

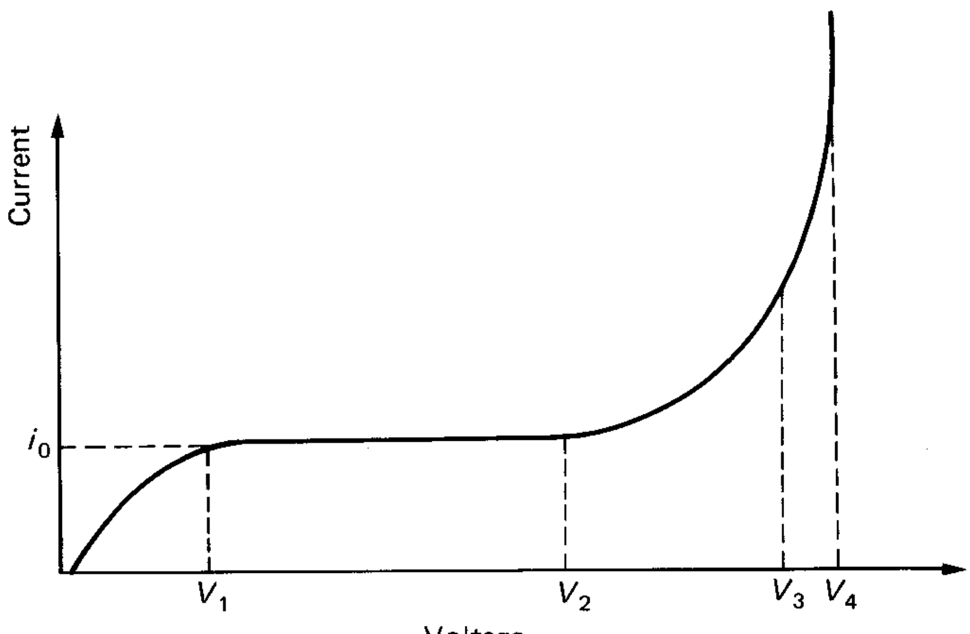


Figure 6.1.1.1: Relationship between current and voltage in pre-spark region[38]

Townsend proposed that the exponential increase in current was due to a process known as "ionisation by collision". Townsend defined α, known as Townsend's first ionisation coefficient, as "the number of electrons produced by an electron per unit length of path in the direction of the field"[39]

Assume that an electron creates α new electrons in 0.01 m in the direction of the field. Then the increase in the number of electrons produced by *n* electrons over a distance *dx* is:

$$dn = \alpha n dx \qquad \text{Equation (6.1.1.1)}$$

If we integrate over the total distance from anode to cathode (d) we get:

$$n = n_0 e^{\alpha x} \qquad \text{Equation (6.1.1.2)}$$

where n_0 is the initial number of electrons and $e^{\alpha d}$ is known as the electron avalanche.

If we write the same Equation in terms of current we have:

$$I = I_0 e^{\alpha d} \qquad \text{Equation (6.1.1.3)}$$

where I_0 is the current at the cathode.

The electron avalanche represents the increased number of electrons, thus greatly increasing the available current carriers.

6.1.2. Photoionisation

If an ion with energy less than that required for ionisation collides with another particle it may lead to excitation of that particle. When the particle drops back into its usual state it emits the energy of a photon (hv). This emitted energy can then ionise another particle. This process is known as "photoionisation". If the photon emitted has energy less than that required to cause ionisation then when it collides with another particle it may lead to photo excitation. Photoionisation is a secondary process and it plays an essential part in the streamer breakdown process and some corona discharges.

6.1.3. Ionisation by collision of metastables with atoms

Some excited states may last for up to a few seconds. These are known as metastable states and metastable atoms have relatively high potential energies. If the metastable atom has energy greater than that required to ionise some particle then the collision results in ionisation. If the metastable atom has energy less than that required for ionisation, then collision with another particle will result in excitation of that particle.

6.1.4. Secondary Ionisation Processes

Normally electrons are held in a solid by electrostatic forces. However if sufficient energy is supplied these forces can be overcome and electrons can break free. The energy required to overcome the electrostatic forces varies for each material and is called the work function, represented by W_a. Electrons released from the tip of the lightning rod in this manner are important for initiating and sustaining an upward leader.

6.1.5. Photoelectric emission

If a photon collides with the surface of a material and the energy of that photon is greater than the work function of the material, i.e. $hv > W_a$, then the excess energy can be transferred to the kinetic energy of an electron which is ejected from the material. This process is described by Einstein's relation in Equation (6.1.5.1) below.

$$\frac{1}{2}mu_e^2 = hv - hv_0 \qquad \text{Equation (6.1.5.1)}$$

where m is the mass of the electron,

u_e is the velocity of the electron,

hv is the energy of the incident photon and

hv_0 is equal to the work function W_a of the material.

This process is subject to the probability that the energy of the photon is greater than the work function of the material. Therefore the probability of getting a photon is the probability that $hv > W_a$.

6.1.6. Emission by positive ion and excited atom impact

Bombardment of the surface of a material by positive ions or metastable atoms can also result in the emission of electrons. When the surface is bombarded by positive ions there are in fact two electrons ejected, one neutralises the charge on the positive ion and the other is emitted.

6.1.7. Field Emission

It is possible for a strong electric field to draw electrons from the surface of a material. The fields required for this to occur are in the range of 10^7 - 10^8 V/cm. These strengths are easily reached around the sharp tip of a lightning rod. This process of electron emission from a surface is known as the tunnel effect.

6.1.8. Townsend's second ionisation coefficient γ

Townsend observed the current measured at very high voltages did not fit with his theoretical expectations. The current measured should be given by the Equation (6.1.1.3).

Therefore a graph of log(I) versus the width of the gap should give a linear relationship. However Townsend found that at high voltages the measured current did not follow this relationship, and increased more quickly than he had predicted.

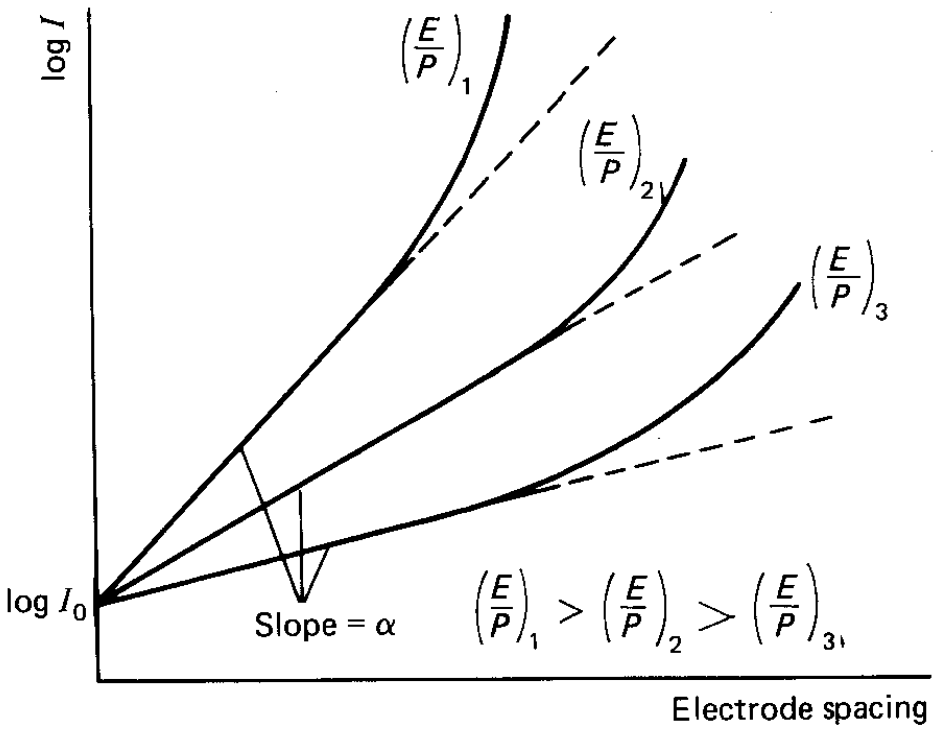


Figure 6.1.8.1: Variation of gap current with electrode spacing in uniform field gaps.[40]

To describe this deviation Townsend introduced a coefficient γ known as Townsend's second ionisation coefficient. He proposed that this increase in current was due to secondary effects such as:

- The liberation of electrons in the gas by collision of positive ions.
- The release of electrons from the surface of the material due to bombardment by ions and metastable atoms (as discussed).
- Secondary emission due to the impact of photons with the surface of the material.
- Photoionisation of the gas.

Townsend hypothesised that all of these secondary effects lead to an increase in current at higher voltages. The second ionisation coefficient, γ, is used to account for a combination of these secondary effects.

6.2. Decay Processes

6.2.1. Recombination

Recombination will always occur when there are positive and negative ions present. The potential and relative kinetic energies of the recombining electron and positive ion are released upon recombination as a quantum of radiation. This process can be represented as follows:

$$A^{+} + e \rightarrow A + hv$$ [41] Equation (6.2.1.1)

where A^{+} represents the positive ion, e represents the electron, and hv represents the quantum of radiation.

It is also possible for a third particle or electron, C, to be involved which absorbs the energy that is released during the recombination process. This is represented by:

$$A^{+} + C + e \rightarrow A^{*} + C \rightarrow A + C + hv$$ Equation (6.2.1.2)

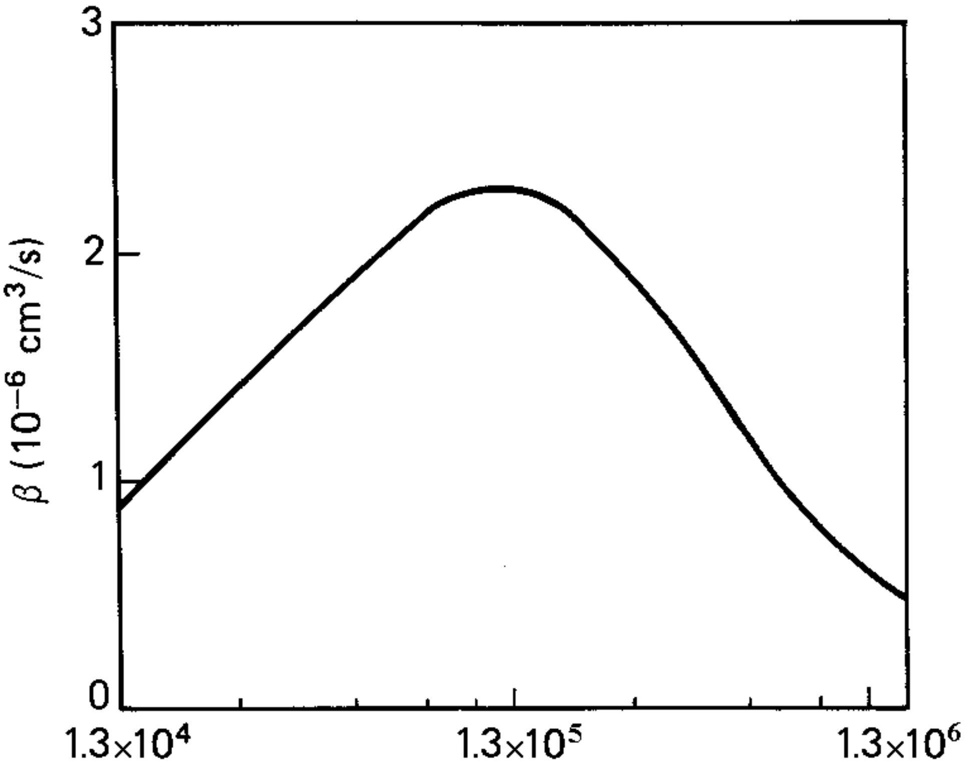
where A^{*} represents the atom in an excited state.

The rate of recombination is proportional to the concentration of positive and negative ions. The rate of recombination can be expressed as below:

$$\frac{dn_{+}}{dt} = \frac{dn_{-}}{dt} = -\beta n_{-} n_{+}$$ [42] Equation (6.2.1.3)

where n_{+} and n_{-} are the concentrations of positive and negative ions respectively.

The proportionality constant β is the recombination rate coefficient. The coefficient β varies with pressure.



p (Pascals)

Figure 6.2.1.1: Variation of recombination coefficient with pressure in air at 20 ^{0}C.[43]

The recombination process is important at high pressures whereas diffusion is relatively unimportant.

6.2.2. Deionisation by attachment

Electronegative gases are gases that have an outer shell that has only one or two electrons. Therefore these gases readily accept another electron and form a stable negative ion. When the electron attaches itself to an atom a quantity of energy, known as the electron affinity of the atom, is released.

For an electron moving with a drift velocity of μE cm/sec, where μ is the mobility of the particle and E is the electric field strength, v_c is the number of impacts it makes each second under the influence of an electric field, E. Therefore over a distance of 0.01m the electron makes $v_c / \mu E$ impacts. The attachment process can be represented as shown below:

$$dn = -hn\frac{v_c}{\mu E}dx$$ [44] Equation (6.2.2.1)

where h is the a constant known as the probability of attachment. This Equation can also be written as:

$$n = n_0 e^{-(hv_c/\mu E)x}$$ [45] Equation (6.2.2.2)

There are several processes of negative ion formation and most of these are reversible, therefore leading to electron detachment.

The most significant processes to consider when determining charge density are ionisation by collision, recombination and deionisation by attachment since they have the greatest effect on the generation and decay of charged species. These are the most important processes, but there are many others that will contribute to some degree. Initially these three effects could be modelled and other effects could be included later to see what effect they have, if any.

Some of these processes would produce only very small numbers of charged particles and therefore their inclusion in the model would serve no real purpose. It would also be time consuming and introduce more possible sources of error for little return.

7. MATHEMATICAL MODELLING

The parameters chosen were the same as last years model where possible so that comparisons could be made between the two and three - dimensional models.

7.1. Initial Voltage Distribution

The initial voltage distribution was 15 kVm^{-1}. This value could have been anywhere between 15 and 25 kVm^{-1}, but 15 kVm^{-1} was chosen since this was the value that was used last year. The voltage was set to zero volts at the bottom of the area and was increased by 15 kV per metre.

7.2. Solving Poisson and Laplace's Equations

Poisson's Equation is given by:

$$\nabla^2 V = \frac{\rho}{\varepsilon} \qquad \text{Equation (7.2.1)}$$

Where: V = potential (V)

ρ = charge density (Cm^{-1})

ε = permittivity of medium (Fm^{-1})

In cylindrical coordinates Poisson's Equation is written as:

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2} = \frac{\rho}{\varepsilon} \qquad \text{Equation (7.2.2)}$$

Poisson's Equation is used to solve for the electric field once the corona effects have been taken into account.

Laplace's Equation is a special case of Poisson's Equation, for a region that is free of charge, i.e. $\frac{\rho}{\varepsilon} = 0$

In cylindrical coordinates Laplace's Equation is written

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2} = 0 \qquad \text{Equation (7.2.3)}$$

The Finite Difference Method was used to solve these Equations for the voltage at a point. To obtain the solution we consider the potential distribution about a point P.

Let the potential at point P be V_0 and the potentials of the surrounding points V_1, V_2, V_3, V_4, V_5, and V_6 as shown below.

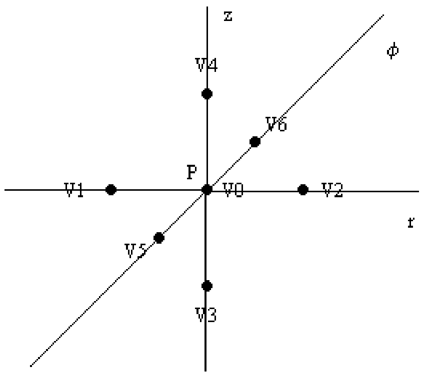


Figure 7.2.1: Voltage points surrounding P_0.

Consider each term of Poisson's Equation separately.

$$\frac{\partial^2 V}{\partial r^2} \approx \frac{\left[\frac{(V_2 - V_0)}{\Delta_r} - \frac{(V_0 - V_1)}{\Delta_r}\right]}{\Delta_r}$$

$$= \frac{V_2 - 2V_0 + V_1}{{\Delta_r}^2}$$

$$\frac{\partial^2 V}{\partial z^2} \approx \frac{\left[\frac{(V_4 - V_0)}{\Delta_z} - \frac{(V_0 - V_3)}{\Delta_z}\right]}{\Delta_z}$$
$$= \frac{V_4 - 2V_0 + V_3}{\Delta_z{}^2}$$

$$\frac{\partial^2 V}{\partial \varphi^2} \approx \frac{\left[\frac{(V_6 - V_0)}{\Delta_\varphi} - \frac{(V_0 - V_5)}{\Delta_\varphi}\right]}{\Delta_\varphi}$$
$$= \frac{V_6 - 2V_0 + V_5}{\Delta_\varphi{}^2}$$

Therefore Poisson's Equation becomes:

$$\frac{V_2 - 2V_0 + V_1}{\Delta_r{}^2} + \frac{1}{r}\frac{(V_2 - V_1)}{2\Delta_r} + \frac{1}{r^2}\frac{(V_6 - 2V_0 + V_5)}{\Delta_\varphi{}^2} + \frac{(V_4 - 2V_0 + V_3)}{\Delta_z{}^2} = \frac{\rho}{\varepsilon}$$

Equation (7.2.4)

A square grid has been chosen and therefore $\Delta_r = \Delta_z = \Delta$. However Δ_φ can be varied, and is defined in the program.

Therefore the above Equation can be rearranged to give V_0:

$$V_0 = \frac{r^2.\Delta_\varphi^2(V_1 + V_2 + V_3 + V_4) + \frac{\Delta.r.\Delta_\varphi^2}{2} + \Delta^2(V_5 + V_6)}{4.r^2.\Delta_\varphi^2 + 2.\Delta^2}$$

Equation (7.2.5)

7.2.1. Solving for unevenly spaced grid

The above Equation can only be used where the grid spacing between each point is constant. We must also be able to solve for the case where a boundary passes between two points as shown below.

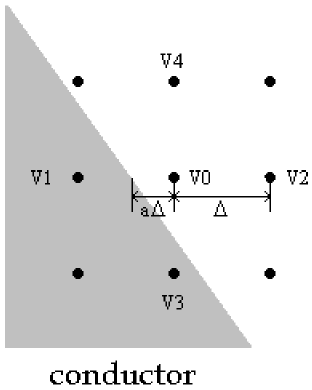


Figure 7.2.1.1: Boundary between grid points.

The double derivatives in the z and φ directions are unaffected.

However solving in the r direction gives:

$$\frac{\partial^2 V}{\partial r^2} = \left[\frac{2a}{a+1}\right]\left[\frac{\frac{(V_2 - V_0)}{\Delta} - \frac{(V_0 - 0)}{a\Delta}}{\left(\frac{a+1}{2}\right)\Delta}\right]$$

$$= \left[\frac{4a}{(a+1)^2}\right]\left[\frac{V_2 - \left(1 + \frac{1}{a}\right)V_0}{\Delta^2}\right]$$

$$= \frac{\left[\frac{4a}{(a+1)^2}\right]V_2 - \left[\frac{4}{a+1}\right]V_0}{\Delta^2}$$

Similarly we get:

$$\frac{\partial V}{\partial r} = \left[\frac{2a}{a+1}\right]\left[\frac{(V_2 - 0)}{\Delta}\right]$$
$$= \left[\frac{2a}{(a+1)^2}\right]\left[\frac{V_2}{r\Delta}\right]$$

Therefore, for this more general case of Laplace’s Equation we get:

$$\nabla^2 V = \left(\frac{\left[\frac{4a}{(a+1)^2}\right]V_2 - \left[\frac{4}{a+1}\right]V_0}{\Delta^2}\right) + \left(\frac{1}{r}\left[\frac{2a}{(a+1)^2}\right]\left[\frac{V_2}{\Delta}\right]\right) + \left(\frac{1}{r^2}\left[\frac{V_6 - 2V_0 + V_5}{\Delta^2{}_{\varphi}}\right]\right) + \frac{V_4 - 2V_0 + V_3}{\Delta^2}$$
$$= 0$$

$$\therefore V_0 = \frac{\left[\frac{2.a.V_2(2.r+\Delta)}{r(a+1)^2} + \frac{\Delta^2(V_5 + V_6)}{r^2.\Delta^2_{\varphi}} + V_3 + V_4\right]}{\left[\frac{4}{(a+1)} - \frac{2.\Delta^2}{r^2.\Delta^2_{\varphi}} + 2\right]} \qquad \text{Equation (7.2.1.1)}$$

7.3. Finite - Difference Method (FDM)

Finite - difference methods can be used to obtain a numerical solution for any steady – state or time varying two or three – dimensional field problem. Finite – difference methods can be used regardless of the shape of the boundary, the boundary conditions, or the quantity of interconnected regions.[46] Their application only involves simple arithmetic and is therefore very easy. The main disadvantage of finite-difference methods, as with all numerical methods, the solution must be performed for every set of parameters defining a problem.[47]

The boundary conditions of the given area must be known. The solution is obtained by replacing the partial differential Equation for the region, in this case Laplace or Poisson’s Equation, by a number of finite difference Equations. Therefore the solution of the field is reduced to a set of simple Equations giving the potential at the grid points. This set of Equations is then solved iteratively to a specified degree of accuracy.

For bigger problems machine computation is necessary, and iterative schemes are most efficient in combination with successive over-relaxation methods.[48]

Each potential and its distribution within the region of interest is continuous in nature. However each computation can only provide a limited amount of information, therefore a discretisation of the area is needed to provide nodes for which a solution can be found.

Superimposing a grid with regular nodes over the region produces the required nodes for which a solution can be found. This mesh is usually made up of regular grids or polygons e.g. squares, rectangles, triangles or hexagons. The most commonly used polygons are squares (or rectangles) and equilateral triangles. When selecting the shape of the grid if possible choose a coordinate system so that the boundaries can be simply specified. It is also important to choose a grid that is not too fine as this results in unnecessary computation and poor convergence.

Often the problem to be solved is very large and so it is not practical to initially define size of the grid to be as small as desired. It is usual to start with a coarse grid until it has converged to a specified value and then make the grid finer. It is usually convenient to reduce the grid size by half so that the nodes of the new grid lie midway between the previous nodes. The potential at the new nodes is calculated using linear interpolation. Continue to make the grid successively finer until the desired grid size is reached. Of course it is possible to make the grid finer only in certain regions of interest.

The process of iteration can be accelerated by applying a relaxation factor to each node. It is also faster if an initial estimate is made of the value of each point.

When the iterative solution is calculated by employing a computer the most popular method for rapid convergence is the successive over relaxation method (SOR). This is the method employed in this project. The successive over relaxation method was first described by Frankel and Young independently of each other.[49] The successive over relaxation method is the most flexible of the convergent iterative methods which means that it is one of the most popular methods.[50] It is employed by applying an over relaxation factor point by point.

The successive over relaxation method is derived from the Gauss – Seidel method. The new value is calculated as described below:

$$new\ value = old\ value + \alpha\left[new\ value - old\ value\right]$$ [51]

Where α is the convergence or relaxation factor. The value of α must lie between one and two. If $\alpha = 1$ then there is no acceleration however, depending on the problem, if $\alpha \geq 2$ then the solution will often be unstable. It its also possible for the solution to become unstable if α is too large even if it is less than two. There is an optimum value for α which is different for each problem. However the optimum value for α can vary from one stage of computation to the next.

A relaxation factor of $\alpha = 1.2$ has been used in this project. This value was experimented with, however when α was increased much beyond this point the solution no longer converged.

There are two possible sources of error when using the finite - difference method. Error can be introduced when the differential Equation for the field is replaced by the set of finite – difference Equations. This is because when the finite – difference Equations are derived higher terms of the Taylor series are omitted which means that they are only an approximation of the original Equation.[52] This error is known as truncation error. There is also an error introduced because the difference Equations are not solved exactly, but only to a certain degree of convergence. Both of these errors have been assumed to be negligible for the calculations in this project.

7.4. Treatment of Boundaries

The treatment of boundaries was one of the major sources of problems in this modelling process. The boundaries at the top, bottom and the outer edge of the modelled area were set to a fixed value. This value was the value assigned to these boundaries after the initial voltage distribution had been allocated.

The boundary of the rod was more complex. A different form of Laplace's Equation was used to calculate the potential at the boundary of the rod. This used a value 'a', which was defined as the distance from the point to the edge of the rod. See Equation(7.2.1.1).

The boundary on the inside edge of the modelled area was not set as fixed since the potential at this boundary will change. However this presented a problem since in solving Laplace's Equation the element [j-1] needs to be accessed which does not exist at j = 0. This was overcome by assigning the value at j = 0 to the point [j–1].

8. THE PROGRAM

The program runs with the desired parameters and produces a data file of the potential at each point. The program was written and compiled using C++ Visual Basic. The output was graphed using Matlab.

The area to be modelled and the distance between the grid points was defined. The voltage distribution was initially set to be 15 kVm^{-1}. 'i' was used as the row coordinate, 'j' was used as the column coordinate, and 'k' was used to define the angular coordinate. This is shown in Figure 8.1 below.

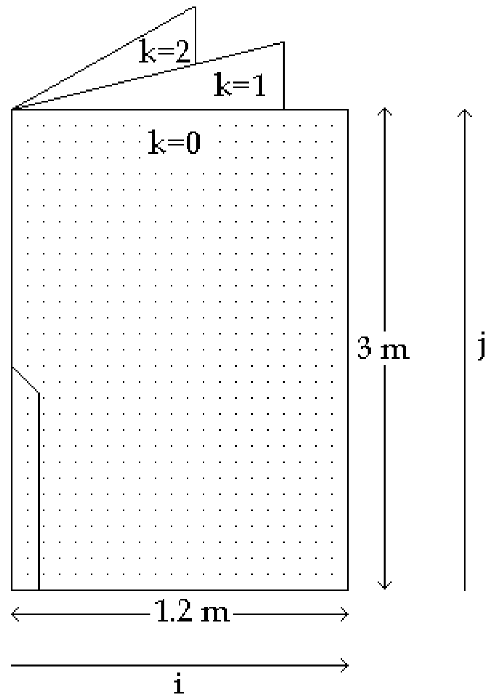


Figure 8.1: Diagram of coordinate and grid system used in program

The program was only run for rough grid, and only two or three angular "slices" due to time constraints and processing time. However once the program is working for a small area it is a simple matter to expand the modelled area.

It was important when writing this program to write efficient code. The iterations are lengthy, depending on the size of the grid and the area modelled, and inefficient code would further exacerbate the problem.

This program does not model the electric field or the charge density. However the flow chart in Figure 8.2 below shows how the iterative procedure can be used to solve for these from the potential distribution.

Most of the calculations in the program have been broken down into simple procedures, most of which should be self-explanatory. Also most of the variable names used should make it clear as to what the variables represent. I have included a brief discussion of some of the more important and complex procedures as well as clearly defining some of the more ambiguous variables. The program is given in the Appendix and is commented throughout. There is also a copy of the program on CD in the back cover.

Header files are used to define variables or procedures that are commonly used throughout the program. The use of a header file means that it is quick and easy to change the values of variables.

The file "parameters.h" is a header file that defines all the global variables used in the program. This file contains all the variables that may need to be changed such as the area modelled, or the height of the rod.

The relaxation factor is the value applied to each point so that the solution converges faster. The value of 1.2 was chosen because if it was increased much beyond this the solution did not converge. See Section 7.3 for more discussion about choosing the relaxation factor.

The converge threshold is the value below which the solution is said to be converged.

The file "MainProgram.h" is also a header file. It contains the headers for all the procedures in the program.

Set the initial voltage distribution to 15 kVm^{-1}

↓

Set the boundary conditions:

At top and outer edge of modelled area = 15 kVm^{-1} x height (m)

At ground level and rod = 0 V

↓

Solve Laplace's Equation to give an estimate of the voltage distribution:

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

↓

Calculate $\underline{E} = -grad(V)$ to give the electric field

↓

Assume a time step Δt then estimate the charge density $\rho(r, z, \varphi)$

↓

Solve Poisson's Equation to give a new estimate of the voltage distribution:

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2} = \frac{\rho}{\ }$$

(return to "Calculate $\underline{E} = -grad(V)$")

Figure 8.2: Flow chart showing iterative process of solving problem

‘initialiseRadiusVector’

This initialises a vector that stores the radius from the centre of the rod in metres to the point specified by the variable ‘j’. The radius vector is used when solving Laplace’s Equation.

‘initialiseGeometry’

This initialises the array ‘geometry[row][column][angle]’ which stores either ‘0’ or ‘1’ for each point. A ‘0’ means that the point is not fixed, and its value can be changed during the course of the program. However a value of ‘1’ means that the point is fixed and therefore it has the same value throughout the program. The values of the fixed points are set during the initial voltage distribution procedure.

Initially all the geometry points are set to zero (not fixed). Then the points at the top, bottom and the outer edge are set to be fixed. This means that these boundaries are not changed when Laplace’s Equation is being solved. Laplace’s Equation requires fixed boundaries.

‘initialiseVolts’

This sets the voltage equal to zero inside the rod and 15 kVm^{-1} for the rest of the region. It also sets the geometry to be fixed inside the rod. It makes use of the function ‘Rod_geometry(i)’ to determine whether the grid point is inside the boundary of the rod.

‘Rod_geometry’

The procedure ‘Rod_geometry’ calculates the radius of the rod as a function of height. The radius at the tip is calculated by using the formula for a line $y = mx + c$. Using this formula the distance from the centre of the rod to the edge at the tip is calculated. For the main part of the rod the procedure simply returns the radius of the rod in metres.

8.1 Solving Laplace's Equation

Laplace's Equation is solved iteratively until the solution has converged. The number of iterations taken for the solution to converge is counted and displayed to the screen.

Laplace's Equation was solved using Equations (7.2.5) and (7.2.1.1) respectively, depending on whether the point was near a boundary or in free space.

'solveSingleIteration'
The voltages at the boundary points were set so that the element of the array that does not exist for each boundary was set to be the value at the boundary point. For example, at the boundary where i = 0, the element [i-1] does not exist therefore the element [i-1] is assigned the value of element [i] for this case. Similarly for the boundaries where i = row (the maximum value of i) and for j = 0 and j = column (the maximum value of j).

For the boundary where k = 0 the [k-1] element was assigned the value of k at k = angle (the maximum value of k), and where k = angle the [k+1] element was assigned to the value at k = 0.

Only if the geometry is equal to '0' (not fixed) will a new voltage be calculated for the point.

If the geometry at the point is such that the points to the left and below left both have geometries equal to one, then the point is near the boundary of the rod and the new voltage is calculated using 'calculateLaplaceForFixed'. Both these points are tested because once the program includes the charge effects the area between grid points will be used to determine flow of charge, not just the points.

If the point is not near a boundary then the new voltage is calculated using 'calculateLaplaceForFree'.

For each point the difference between the new voltage and the previous voltage is calculated. Then the new voltage becomes equal to the previous voltage add the product of the difference and the relaxation factor. Using the relaxation factor makes the solution converge more quickly. See Section 7.3 for further discussion on the relaxation factor.

If the difference between the new and previous voltages is greater than 'biggest_change' then 'biggest_change' becomes equal to the difference. When the value of 'biggest change' becomes less than the converge threshold the solution has converged.

The value of 'biggest_change' is outputted to the screen every 50 iterations so that the user can see the solution converging.

'getA'
This procedure returns the distance the grid point is from the edge of the rod at the boundary of the rod.

'calculateLaplaceForFixed'
This procedure uses Laplace's Equation for an unevenly spaced grid (Equation (7.2.5)) to solve for the new voltage.

'calculateLaplaceForFree'
This procedure uses Laplace's Equation (Equation (7.2.1.1)) to solve for the new voltage.

8.2 Graphing Output

The C++ program output data to a data file and an Excel spreadsheet. The graphing functions in Excel were not very appropriate for such large arrays or three–dimensional problems. Therefore the potential distribution was displayed graphically using Matlab.

A short piece of code was written in Matlab to read the data stored by the C++ program and plot it. The commented code is given in the Appendix.

9. FURTHER WORK

This project provides a basis on which further work can be done. The three-dimensional model is in place and from this it is possible to solve for the electric field and ultimately include the effects of wind.

If further work is to be undertaken, it is recommended that B. Cridland's program be read as it provides a useful basis from which to work. The program is only two–dimensional, but the modelling of the processes can still be applied and altered to include the third dimension.

This program only models a small area. If a larger more powerful computer was available it would be possible to model larger areas.

It may also be useful to investigate the possibility of using Matlab rather than C++ to write the program as it is more suited to the handling of large arrays and complex mathematics.

A slightly different way of approaching the problem that should eliminate the boundary error would be to model the area with the rod in the middle rather than at the edge. This would require some changes to the code, but it would be very similar.

10. Results

The potential distribution as modelled by my program is shown in Figure 10.1 below. In this case, where there is no charge being accounted for, the potential distribution is the same for every angle around the rod.

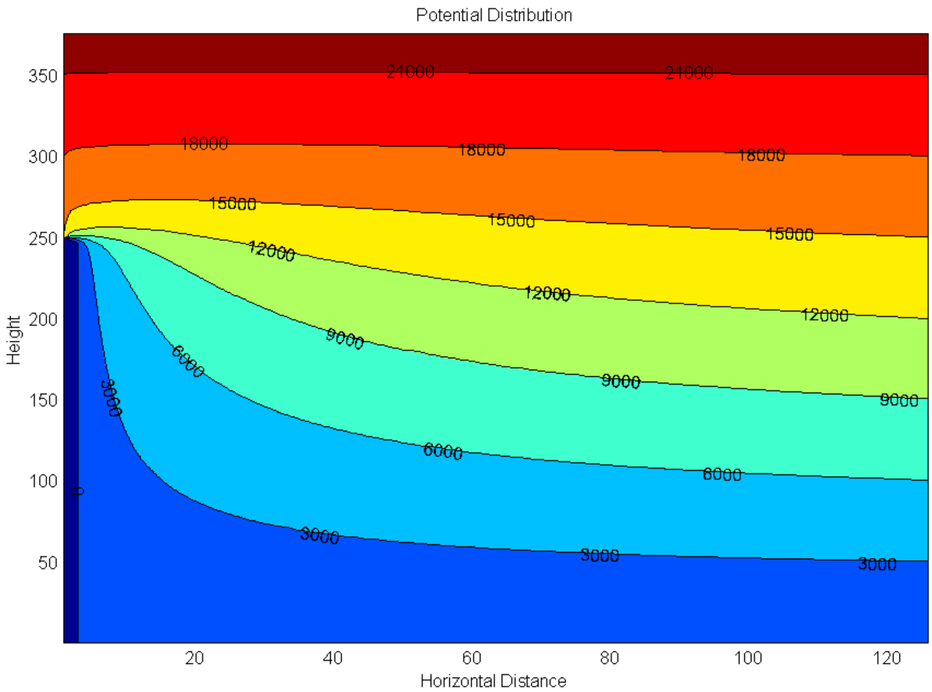


Figure 10.1: Output from program of electric field around lightning rod.

From this graph we can see that the program is not modelling the potential distribution correctly around the tip of the rod. This error is most likely from boundary conditions. The electric field should not hook into the tip of the rod, but should curve over the top as shown in Figure 10.2 below.

I cannot find the source of this error. It is most likely due to the boundary conditions but the exact problem could not be pinpointed.

Apart from this error the program models the potential distribution accurately around the rod.

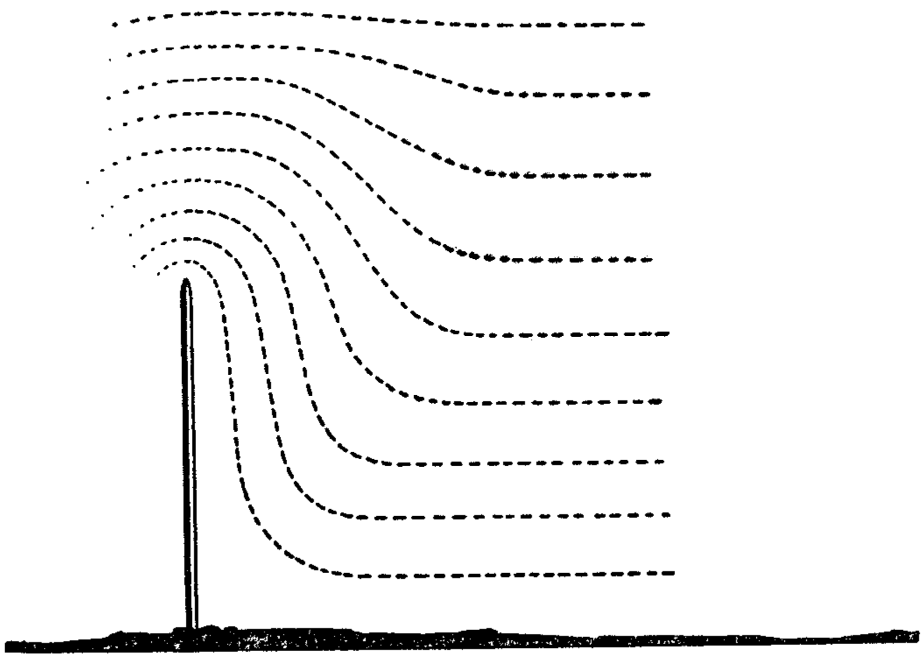


Figure 10.2: Potential distribution around lightning rod.[53]

11. Conclusion

The aim of this project was to model the field around a lightning rod in three-dimensions. The potential distribution was modelled, but the model was not accurate. Near the tip of the rod there is distortion of the potential due to an error in the boundary conditions.

The program provides a basis from which further work can be done. From this program the three-dimensional electric field can also be calculated. Once the charge species have been accounted for wind can be included in the model and its effects on the electric field can be investigated.

The project provides a framework for the three-dimensional problem that can be expanded on.

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